# Machine Learning Through Mathematics

**Week 1 Exercise Sheet: AI, Machine Learning, and Learning Types**

## Brief recap

- **Artificial intelligence (AI)** is the broad idea of machines doing tasks that seem intelligent.
- AI can be **rule-based** and does not always learn.
- **Machine learning (ML)** is a part of AI where a computer learns patterns from data.
- A simple mathematical way to think about ML is:

$$f(x) \approx y$$

- The three main types of ML are supervised, unsupervised, and reinforcement learning.

## Exercises

### Comparing systems

Consider two systems:

- System A: uses fixed rules written by a human
- System B: learns from data but makes small errors

a) Give one advantage of System A.

b) Give one advantage of System B.

c) Explain why System B might still be preferred in real-world problems.

### Limits of learning

Answer concisely but carefully.

a) Why can a model that performs perfectly on training data still fail in practice?

b) What role does the amount of data play in learning?

c) Give one example where more data would **not** fix the problem.

### Vectors as inputs

A machine learning model takes a 3-dimensional input vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and predicts

$$\hat{y} = x_1 + 2x_2 - x_3$$

a) Compute $\hat{y}$ for

$$\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$$

b) Find a vector $\mathbf{x}$ such that $\hat{y} = 0$.

c) Describe (in words) what kind of inputs make $\hat{y}$ large.

d) Two students have vectors $\mathbf{x}_A$ and $\mathbf{x}_B$. Explain what $\frac{1}{2}(\mathbf{x}_A + \mathbf{x}_B)$ represents in this context.

### Data quality and model behaviour

A model is trained on the following (x,y) data:

$$(1, 2), (2, 4), (3, 6), (4, 8)$$

a) Suggest a simple function $f(x)$ that fits this data.

b) Predict the output for $x = 10$.

c) Now suppose the true relationship is actually noisy in reality. Explain why this model might fail on new data.

d) What would happen if one incorrect point $(3, 100)$ were added? Explain the possible effect on the learned model.

### Synthesis challenge

You are designing a system to recommend videos.

a) Define a possible input vector $\mathbf{x}$ (at least 3 features).

b) Define the output $y$.

c) State the learning type.

d) Explain one limitation of your design.